Modelling Technical and Environmental Efficiency

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Abstract

In agricultural studies, stochastic frontier models are commonly used for measuring technical efficiency of an individual farm. In this paper, we generalise these models to allow for environmental inefficiency. We begin by reviewing standard stochastic frontier models where the farm produces a single output. We then generalise to the case where the farm produces several good outputs. The final model we consider has both good and bad outputs. In empirical studies, the latter are usually environmental pollutants. We discuss how both technical and environmental efficiency can be measured in the context of this model. These methods are applied to a data set involving Dutch dairy farms. The data set includes both good outputs (milk and non-milk output) and a bad output (a measure of Nitrogen surplus).

Introduction

The environmental problems caused by modern agriculture present an increasingly worrying policy problem. In the application used in this paper, Dutch dairy farms produce not only good outputs, such as milk, but also undesirable outputs, such as excessive nitrogen due to the application of manure and chemical fertilizers. It is thus important to understand the nature of the best practice technology available to farmers for turning inputs into good and bad outputs. Furthermore, it is important to see how individual farmers measure up to this technology. In other words, evaluation of farm efficiency, both in producing as many good outputs and as few undesirable outputs as possible, is crucial. In this paper, we describe how extensions of stochastic frontier models can be used to shed light on these issues. We begin by surveying the standard stochastic frontier model with one output to make concrete the basic ideas of efficiency analysis. We then generalise to allow for several good outputs. Next, we consider the case with several outputs, where some of them can be undesirable. We then present some of the results for our empirical application involving Dutch dairy farms.

The Stochastic Frontier Model with a Single Output

Stochastic frontier models are commonly used in the empirical study of farm[1] efficiency

[1] We will use the term "farm" to refer to the cross-sectional unit of analysis. In practice, it could also be the firm, individual or country, *etc.*

and productivity. The seminal papers in the field are Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977), while a recent survey is provided in Bauer (1990). The ideas underlying this class of models can be demonstrated using a simple production model[2] where output of farm i, Y_i, is produced using a vector of inputs, X_i, $(i = 1 \ldots N)$. The best practice technology for turning inputs into output depends on a vector of unknown parameters, β, and is given by:

$$Y_i = f(X_i; \beta). \quad (1)$$

This so-called production frontier captures the maximum amount of output that can be obtained from a given level of inputs. In practice, actual output of a farm may fall below the maximum possible. The deviation of actual from maximum output is a measure of inefficiency and is the focus of interest in many applications. Formally, equation (1) can be extended to:

$$Y_i = f(X_i; \beta)\tau_i, \quad (2)$$

where $0 \leq \tau_i \leq 1$ is a measure of farm-specific efficiency and $\tau_i = 1$ indicates farm i is fully efficient.

The model given is equation (2) implicitly assumes that all deviations from the frontier are due to inefficiency. However, following standard econometric practice, we add a random error to the model, ζ_i, to capture measurement (or specification) error, resulting in:

$$Y_i = f(X_i; \beta)\tau_i\zeta_i. \quad (3)$$

The addition of measurement error makes the frontier stochastic, hence the term "stochastic frontier model". We assume that data for $i = 1 \ldots N$ farms is available. It is common to assume that the production frontier, $f(\cdot)$, is log-linear (*e.g.* Cobb-Douglas or translog). We define X_i as a $1 \times (k+1)$ vector (*e.g.* $X_i = (1 \; L_i \; K_i)$ in the case of a Cobb-Douglas frontier with two inputs, L and K) and, hence, (3) can be written as:

$$y_i = x_i\beta + v_i - z_i, \quad (4)$$

where $\beta = (\beta_0 \ldots \beta_k)'$, $y_i = ln(Y_i)$, $v_i = ln(\zeta_i)$, $z_i = -ln(\tau_i)$ and x_i is the counterpart of X_i with the inputs transformed to logarithms. z_i is referred to as inefficiency and, since $0 \leq \tau_i \leq 1$, it is a non-negative random variable. We assume that the model contains an intercept with coefficient β_0. Equation (4) looks like the standard linear regression model, except that the "error" is composed of two parts. This gives rise to another name for these models, *viz.* "composed error models".

The econometric estimation of this model can be done using classical or Bayesian approaches. In previous work, we have argued for the advantages of a Bayesian approach to surmount some difficult issues in classical econometrics. An introductory survey chapter, Koop and Steel (1999), discusses Bayesian estimation and computation in detail for this model. Classical econometric methods are discussed in Horrace and Schmidt (1996). For reasons of brevity, econometric estimation will not be discussed in the present paper. For all cases relating to single-output production frontiers (e.g. with cross-sectional or panel data, with linear or nonlinear production frontiers, etc.) the reader is referred to Koop and Steel (1999) which is available at http://www.ed.ac.uk/~gkoop/.

The Stochastic Frontier Model with Multiple Good Outputs

In Fernandez, Koop and Steel (1999a), we developed extensions of stochastic frontier models to allow for efficiency analysis in the presence

[2] The discussion here focusses on production frontiers. However, by suitably redefining Y and X, the methods can be applied to cost frontiers.

of multiple outputs. Note that previous work with multiple outputs has often involved either having data on prices (e.g. in order to estimate a demand system) or on costs (e.g. in order to estimate a cost function). However, particularly in the case when some of the outputs are not sold in markets (e.g. pollution), such price or cost information is not available. Hence, it is important to develop methods which involve only output and input data.

The theoretical starting point in most analyses of multiple-output technology is a transformation function:

$$f(y, X) = 0,$$

where y is now a vector of p good outputs and X is a vector of inputs. If the transformation function is separable then we can write it as:

$$\theta(y) = h(X).$$

In the present paper, we assume a constant elasticity of transformation form for $\theta(y)$, but the basic ideas extend to any form.

To establish some terminology, note that $\theta(y) = \textit{constant}$ maps out the output combinations that are equivalent. Hence, it is referred to as the production equivalence surface, which is $(p-1)$-dimensional. By analogy with the single output case, $h(X)$ defines the maximum output (as measured by $\theta(y)$) that can be produced with inputs X and is referred to as the production frontier. If measurement error did not exist, all farms would lie on or within the frontier and deviations from the frontier would be interpreted as farm-specific inefficiency. In essence, $\theta(Y)$ can be treated as a kind of aggregate output. Once $\theta(Y)$ is modelled, it can be used in place of the single output in equation (3) and standard single output methods of efficiency analysis can be used. This strategy is formalized in the remainder of this section.

Since the empirical application used in the present paper involves panel data, we assume a set of NT observations corresponding to outputs of N different farms over T time periods is available. The output of farm i $(i = 1, \ldots, N)$ at time t $(t = 1, \ldots, T)$ is p-dimensional and is given by the vector $y_{(i,t)} = (y_{(i,t,1)}, \ldots, y_{(i,t,p)})' \in \Re^p_+$. We use the following transformation of the p-dimensional output vector:

$$\theta_{(i,t)} = \left(\sum_{j=1}^{p} \alpha_j^q \, y_{(i,t,j)}^q \right)^{1/q}, \tag{5}$$

with $\alpha_j \in (0,1)$ for all $j = 1, \ldots, p$ and such that $\sum_{j=1}^{p} \alpha_j = 1$ and with $q > 1$. For fixed values of $\alpha = (\alpha_1, \ldots, \alpha_p)'$, q and $\theta_{(i,t)}$, (5) defines a $(p-1)$-dimensional surface in $\Re^p_+$ corresponding to all the p-dimensional vectors of outputs $y_{(i,t)}$ that are technologically equivalent. In other words, (5) plots the production equivalence surface. Note that, regardless of the value of q, the production equivalence surface in (5) always intersects with the axes at the same point, namely $y_{(i,t,j)} = \theta_{(i,t)}/\alpha_j$, for all $j = 1, \ldots, p$ and all other outputs equal to zero. Thus, $\theta_{(i,t)}$ is easily visualized as the maximum amount produced of each output multiplied by its corresponding α_j.

Given the transformation from the multivariate output vector $y_{(i,t)}$ to the univariate quantity $\theta_{(i,t)}$ (the parameters of which we estimate from the data), the basic problem of finding farm-specific efficiencies is essentially the same as in the single-output case. If we interpret the value $\theta_{(i,t)}$ as a kind of "aggregate output", then it is sensible to group these transformed outputs in an NT-dimensional vector

$$\log \theta = (\log \theta_{(1,1)}, .., \log \theta_{(1,T)}, .., \log \theta_{(N,T)})', \tag{6}$$

and model $\log \theta$ through the following stochastic frontier model:

$$\log \theta = V\beta - Dz + \sigma\varepsilon. \tag{7}$$

In the latter equation, $V = (v(X_{(1,1)}), \ldots, v(X_{(N,T)}))'$ denotes an $NT \times k$

matrix of exogenous regressors, where $v(X_{(i,t)})$ is a k-dimensional function of the inputs $X_{(i,t)}$ corresponding to farm i ($i = 1, \ldots, N$) at time t ($t = 1, \ldots, T$). The particular choice of $v(\cdot)$ defines the specification of the production frontier: *e.g.* $v(X_{(i,t)})$ is the vector of all logged inputs for a Cobb-Douglas technology, whereas a translog frontier also involves squares and cross products of these logs. The corresponding vector of regression coefficients is denoted by $\beta \in \mathcal{B} \subseteq \Re^k$. Often, theoretical considerations will lead to regularity conditions on β, which will restrict the parameter space $\mathcal{B}$ to a subset of $\Re^k$, still k-dimensional and possibly depending on X. For instance, we typically want to ensure that the marginal products of inputs are positive.

Technological inefficiency is captured by the fact that farms may lie below the frontier, thus leading to a vector of inefficiencies $\gamma \equiv Dz \in \Re_+^{NT}$, where D is an exogenous $NT \times M$ ($M \leq NT$) matrix and $z \in \mathcal{Z}$ with $\mathcal{Z} = \{z = (z_1, \ldots, z_M)' \in \Re^M : Dz \in \Re_+^{NT}\}$. Through different choices of D, we can accommodate various amounts of structure on the vector γ of inefficiencies. For instance, taking $D = I_{NT}$, the NT-dimensional identity matrix, leads to an inefficiency term which is specific to each different farm and time period. $D = I_N \otimes \iota_T$, where ι_T is a T-dimensional vector of ones and $\otimes$ denotes the Kronecker product, implies inefficiency terms which are specific to each farm, but constant over time (*i.e.* "individual effects"). In our application we make the latter choice for D. Since we are working in terms of $\log \theta$, the log of the aggregate output, the efficiency corresponding to farm i at period t will be defined as $\exp(-\gamma_{(i,t)})$ where $\gamma_{(i,t)}$ is the appropriate element of γ.

Bayesian methods, including a discussion of prior elicitation and a Markov Chain Monte Carlo algorithm for posterior simulation, are discussed in detail in Fernandez, Koop and Steel (1999a) which is available at http://www.ed.ac.uk/~gkoop/.

The Stochastic Frontier Model with Multiple Good and Bad Outputs

Important issues in environmental policy hinge on multiple output production technologies where some of the outputs are undesirable. For instance, we have data on farms which produce good outputs (e.g. dairy products) for the market and undesirable outputs (e.g. pollutants). We will refer to undesirable outputs as "bad". Efficiency analysis using stochastic frontier models can be used to shed light on practical policy questions. For instance, if we find dairy farms to be environmentally efficient then pollution can only be reduced by reducing production at dairy farms. However, if many dairy farms are highly environmentally inefficient, then by adopting best practice technology pollution can be reduced without harming production of milk.

Since undesirable outputs such as pollution are almost invariably extremely difficult to price, it is crucial to develop methods of econometric inference which are based on the multiple output transformation function. The question arises as to how to adapt the analysis of the previous section to allow for undesirable outputs and both technical and environmental inefficiency. Following Fernandez, Koop and Steel (1999b), we make one particular adaptation which we argue is reasonable. Others are clearly possible, and these are a topic of past and current research. For instance, Koop (1998) and Reinhard, Lovell and Thijssen (1999) assume that undesirable outputs can be treated as inputs. Here, we model the good outputs as in the previous section, but add a second equation for the transformation function of the bad outputs. Environmental efficiency is measured relative to this second transformation function.

If we let b indicate a vector of m bad outputs, the most general description of best practice technology is given by:

$$f(y, X, b) = 0.$$

We assume this transformation function can broken down into:

$$\theta(y) = h_1(X),$$

and

$$\kappa(b) = h_2(y).$$

In other words, the general transformation function can be broken down into two equations involving a "good production equivalence surface" $\theta(y)$, a "good production frontier" $h_1(X)$, an "environmental production equivalence surface" $\kappa(b)$, and an "environmental production frontier" $h_2(y)$. The assumption that the amount of good outputs produced depends on the inputs, while production of bad outputs depends on the amount of good outputs is likely to be reasonable in many cases. If not, modifications of the present model can be done with only slight alterations to the framework developed in Fernandez, Koop and Steel (1999b).

Formally, we begin with the model for the good outputs given in the previous section given by equations (5) and (7). We further let $b_{(i,t)} = (b_{(i,t,1)}, ..., b_{(i,t,m)})'$ be the vector of m bad outputs for farm i in period t. We define the environmental production equivalence surface through the constant elasticity of transformation form:

$$\kappa_{(i,t)} = \left(\sum_{j=1}^{m} \gamma_j^r b_{(i,t,j)}^r \right)^{1/r}, \qquad (8)$$

with $\gamma_j \in (0, 1)$ for all $j = 1, \ldots, m$ and such that $\sum_{j=1}^{m} \gamma_j = 1$ and with $r > 1$.

Environmental inefficiency is measured using a stochastic frontier model with (8) as dependent variable. That is, we define $\log \kappa$ similarly to $\log \theta$ and set

$$\log \kappa = U\delta + Mv + \varepsilon_b$$

$U = (u(y_{(1,1)}), \ldots, u(y_{(N,T)}))'$. U plays a similar role to V in equation (7) and, hence, the particular choice of $u(\cdot)$ defines the specification of the environmental production frontier. Environmental efficiencies are given by $Mv \in \Re_+^{NT}$. M plays an analogous role to D in the previous section and, as discussed above, different choices for these imply different structure on the inefficiencies. In our empirical work we set $M = D = I_N \otimes \iota_T$ which implies each farm has a technical and environmental efficiency which is constant over time.

Bayesian methods, including a discussion of prior elicitation and a Markov Chain Monte Carlo algorithm for posterior simulation, are discussed in detail in Fernandez, Koop and Steel (1999b) which is available at http://www.ed.ac.uk/~gkoop/.

Application to a Panel of Dutch Dairy Farms

We apply the techniques of the previous section to a data set involving N=613 Dutch dairy farms for the years 1991-94. For each farm data on $p = 2$ good outputs (milk and non-milk production), $m = 1$ bad output (nitrogen surplus) and 3 inputs (labour, capital and variable input) is available. Further detail on this data set is given in Fernandez, Koop and Steel (1999b), Reinhard, Lovell and Thijssen (1999) or the University of Wageningen web site, (www.wau.nl/wub/wep/nr9707/wep07_1.htm). Both the good and environmental production frontiers are assumed to take Cobb-Douglas forms. Complete details of the model specification, including a discussion of distributional forms for the errors, are given in Fernandez, Koop and Steel (1999b). The latter paper also has a much more detailed discussion of empirical results.

Table 1 provides empirical results for this data set. Note that the column labelled "-Median" is the posterior median, a common point estimate. The columns labelled "2.5%" and "97.5" are the 2.5% and 97.5% percentiles, respectively of the posterior (i.e. lower and upper points of a 95% posterior density interval). "RTS" means returns to scale, "Tech. Eff." and "Env. Eff." are the technical and environmental efficiencies for a typical or average farm. Our model allows for technical and environmental efficiencies to be correlated with one another and "Eff. Corr." is this correlation.

Table 1: Posterior Results for Dutch Dairy Farm Data Set

	Median	2.5%	97.5%
β_1(Intercept)	-3.637	-3.744	-3.531
β_2(Labour)	0.110	0.016	0.087
β_3(Capital)	0.532	0.504	0.559
β_4(Variable)	0.473	0.014	0.453
RTS (Goods)	1.115	1.0191	1.141
δ_1(Intercept)	3.057	2.977	3.134
δ_2(Milk)	0.896	0.870	0.922
δ_3(Non-milk)	0.088	0.074	0.101
RTS(Bads)	0.983	0.956	1.011
Tech. Eff.	0.778	0.562	0.978
Env. Eff.	0.552	0.356	0.842
Eff. Corr.	0.344	0.148	0.512

All results seem reasonable. Some of the more interesting results are:

- Firms tend to be more efficient technically then environmentally.
- The positive correlation between efficiencies indicates that farms which tend to be less efficient technically also tend to be less efficient environmentally.
- However, there is a large spread of efficiencies across farms, which manifests itself in large differences between the 2.5 and 97.5th percentiles of both Tech. Eff. and Env. Eff..
- Increasing returns to good output production seems to exist, while constant or slightly decreasing returns exists for bad output production.

We hesitate to draw policy conclusions based solely on this one set of empirical results for one model specification. However, to illustrate the types of issues that our model can be used to address, we offer the following comments. The relatively large degree of environmental inefficiency indicates that pollution can be reduced in many farms at little cost in terms of foregone output. That is, if inefficient farms were to adopt best practice technology and move towards their environmental production frontiers, production of pollutants could be reduced at no cost to milk or non-milk production. The positive correlation between the two types of efficiencies indicates that improving environmental efficiency could be associated with improvements in technical efficiency. Hence, policies aimed at improving efficiency (e.g. by educating farmers in best-practice technology) could have large payoffs. Furthermore, the pattern of returns to scale results indicate that larger farms have advantages. Hence, policies which promote rationalization of farms (e.g. encouraging larger farms to purchase smaller farms) could result both in more production of milk and non-milk outputs (due to increasing returns in the good production frontier) and less pollution (due to decreasing returns in the environmental production frontier).

Conclusions

In this paper, we have shown how the standard stochastic frontier model with a single output can be extended to multiple outputs where some of the outputs are undesirable. The model we develop can be used to understand production technologies which produce pollutants. The empirical application to Dutch dairy farms shows the practicality of this approach and highlights the important policy issues which our model can address.

References

Aigner, D.; Lovell, C.A.K. and Schmidt, P. 1977: Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics*, 6, 21-37.

Bauer, P. 1990: Recent developments in the econometric estimation of frontiers. *Journal of Econometrics*, 46, 39-56.

Fernández, C.; Koop, G. and Steel, M.F.J. 1999a: A Bayesian analysis of multiple output production frontiers. manuscript.

Fernández, C.; Koop, G. and Steel, M.F.J. 1999b: Multiple output production with undesirable outputs. manuscript.

Horrace, W. and Schmidt, P. 1996: Confidence statements for efficiency estimates from stochastic frontiers. *Journal of Productivity Analysis*, 7, 257-282.

Koop, G. 1998: Carbon dioxide emissions and economic growth: A structural approach. *Journal of Applied Statistics*, 25, 489-515.

Koop, G.; and Steel, M.F.J. 1999: Bayesian analysis of stochastic frontier models. Forthcoming in B. Baltagi, ed., *Companion in Theoretical Econometrics*, Basil Blackwell, Oxford.

Meeusen, W. and van den Broeck, J. 1977: Efficiency estimation from Cobb-Douglas production functions with composed errors. *International Economic Review*, 8, 435-444.

Reinhard, S.; Lovell, C.A.K. and Thijssen, G. 1999: "Econometric estimation of technical and environmental efficiency: An application to Dutch dairy farms," *American Journal of Agricultural Economics*, 81, 129-153.